
MATHEMATICS

# USING ALGEBRA TILES EFFECTIVELY

## TOOLS FOR UNDERSTANDING

by

**Bettye C. Hall**

Reviewers

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**PRENTICE HALL MATH**

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# Contents

# Introducing a new, hands-on Professional Development package from Prentice Hall.

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With this new package, all educators can better utilize algebra tiles to reach today's students. New users are given the support they need to start using algebra tiles in the classroom—right away. Experienced users are provided with a host of new applications.

### Comprehensive, train-the-trainer package . . .

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- Written-out scripts that are easy to follow
- Activities for various levels
- Visual Aid masters for transparencies
- Ideas for customizing the workshop for middle grades and high school teachers
- Suggestions for additional resources, including Internet site

### . . . written by an expert in the field.

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- Meets a range of classroom needs
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## Two more professional development exclusives from Prentice Hall:

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# Algebra Tiles: Tools for Understanding

## Purposes of the Professional Development Workshop

Today's mathematics teachers face tremendous challenges, including changes in curricula, teaching techniques, and assessment practices. To help teachers rise to these teaching challenges and opportunities, Prentice Hall presents this self-contained "train the trainer" workshop, developed by nationally-recognized and respected mathematics educators. This workshop is designed to aid educators in meeting the diverse needs and expectations of their students.

The introduction of algebra tiles and other manipulatives into the classroom provides mathematics teachers with exciting opportunities to empower students of all learning styles. Through hands-on activities, this workshop will help teachers become familiar with the uses and applications of algebra tiles. New users will become comfortable using algebra tiles in their classrooms. Experienced users will learn new applications. All users will more fully appreciate the ways that these manipulative tools can help them reach a greater percentage of students.

## Components of This Workshop

This workshop contains activities and visual aid masters that are suitable for use by staff developers, department chairpersons, curriculum specialists, or lead teachers in conducting a workshop on using algebra tiles effectively. This workshop also includes a statement regarding the **purpose** of the workshop, the intended **audience**, a list of necessary pre-workshop **materials** and **audio-visual** requirements, as well as suggestions for **room preparation**.

Because of the wide variety of activities provided, middle school, high school, first-year, and veteran teachers will all benefit from this program. The activities can be customized to accommodate different needs and levels of understanding. Trainers can select the activities that best meet their specific time constraints and professional requirements.

You can introduce the session using any of a variety of **ideas** that are presented at the beginning. Next, there is a variety of **activities** designed to address the workshop goals and objectives. At the end of the workshop are **summary ideas** and suggestions for **closing activities.**

The workshop contains masters for making transparencies or black line copies of the **visual aids**.

## Facilitator's Roles and Responsibilities

As facilitator, you should read through the workshop to become familiar with the objectives and activities. If any of the material is new to you, explore it thoroughly. Select the activities that best suit the needs of the educators attending your workshop. Each section and activity includes the suggested amount of time deemed necessary to accomplish the goal of the activity. Create a reasonable schedule for your workshop, being sure to allow time for exploration and questions.

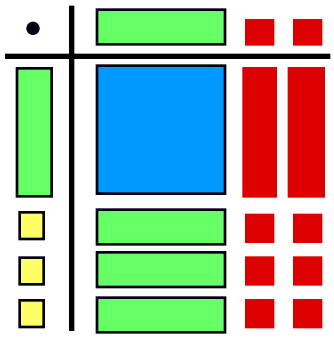
## Suggestions for Getting Started

Introduce yourself. Workshop participants may or may not know each other. If the latter is true, set aside a few minutes for members of the group to get acquainted with one another. Encourage them to talk about their own experiences and their students' experiences using algebra tiles. This should ease participant anxiety.

## Customizing the Workshop

As is always the case, knowledge is best understood and interpreted when it meets the needs of the learner. Knowing the needs of the group to be served, the facilitator will be in the best position to customize the workshop. This workshop, *Using Algebra Tiles Effectively,* contains a sufficient variety of activities so that you can use the workshop with middle grade teachers or high school teachers, educators new to using algebra tiles or educators experienced at teaching with manipulatives and algebra tiles or any combination of these groups. On page 2 of the workshop is a table containing the activities and suggested amount of time necessary to accomplish each activity. Use this table to determine which activities you will use in this workshop.

# Using Algebra Tiles Effectively



$(x + 3)(x - 2) = x^2 + x - 6$

***When I listen, I hear.***
***When I see, I remember.***
***But when I do, then I understand.***

## Purpose

This workshop helps participants use algebra tiles effectively in their classrooms.

## Participants

Middle School and/or High School Teachers

## Materials

- Algebra tiles (1 set for every two participants)
- Flip charts or large pieces of newsprint for recording drawings and number sentences.
- Crayons or colored markers (for each group)

## Audio-Visual Requirements

- Overhead projector and screen
- Algebra tiles for the overhead projector
- Blank transparency film and pens for transparencies
- Transparencies of Visual Aids 1, 2, 3, 4, 5A, 5B, 6A, 6B, 7A, 7B, 8A, 8B, 9A, 9B

## Room Preparation

1. Select desks or tables with flat tops as workspaces.
2. Arrange tables to accommodate groups of two or four.
3. Provide sufficient wall space for posting summaries of group discussions on flip charts or newsprint.

## Pre-Workshop Preparation

Because many teachers were not exposed to manipulative activities as students, they do not use hands-on activities with their classes. In this workshop, the Facilitator acts as the teacher and models the lessons on an overhead projector. The participants act as students and experience the benefits of hands-on activities.

## Workshop Approach

## Workshop Topics

| Activity | Level | Materials | Time |
|---|---|---|---|
| **WORKSHOP OPENING**<br><br>Examining the Algebra Tiles | All Participants | Visual Aid 1 | 10 minutes |
| **ACTIVITY 1**<br>Adding Integers | Middle School Teachers | Tiles: $x$, +1, –1<br>Visual Aid 2 | 10 minutes |
| **ACTIVITY 2**<br>Subtracting Integers | Middle School Teachers | Tiles: $x$, +1, –1<br>Visual Aid 3 | 10 minutes |
| **ACTIVITY 3**<br>Simplifying Algebraic Expressions | Middle School and Algebra I Teachers | Tiles: $x$, +1, –1<br>Visual Aid 4 | 10 minutes |
| **ACTIVITY 4**<br>Solving Linear Equations | Middle School and Algebra I Teachers | Tiles: $x$, +1, –1<br>Visual Aids 5A, 5B | 15 minutes |
| **ACTIVITY 5**<br>Solving Word Problems with Algebra Tiles | Middle School and Algebra I Teachers | Tiles: $x$, +1, –1<br>Visual Aids 6A, 6B | 10 minutes |
| **ACTIVITY 6**<br>Adding and Subtracting Polynomials | Algebra I and Algebra II Teachers | Tiles: $x^2$, $x$, $-x$, +1, –1<br>Visual Aids 7A, 7B | 15 minutes |
| **ACTIVITY 7**<br>Multiplying Polynomials | Algebra I and Algebra II Teachers | Tiles: $x^2$, $-x^2$, $x$, $-x$, +1, –1<br>Visual Aids 8A, 8B | 15 minutes |
| **ACTIVITY 8**<br>Factoring Polynomials | Algebra I and Algebra II Teachers | Tiles: $x^2$, $-x^2$, $x$, $-x$, +1, –1<br>Visual Aids 9A, 9B | 15 minutes |
| **WORKSHOP CLOSING** | All Participants | Recordings from each group | 10 minutes |

Note:
*In addition to the Opening and Closing components,* ***choose at least two activities*** *that best suit your group and your time constraints.*

In this workshop, the Facilitator acts as the teacher and models the lessons on an overhead projector. The participants act as students and experience the benefits of hands-on activities in math classrooms.

## Welcome Participants (5 minutes)

1. After you introduce yourself, discuss the purpose of the workshop:
   - ♦ Say: ***The purpose of this workshop is to help you help your students learn how to use algebra tiles effectively.***
   - ♦ Say: ***Algebra tiles help to develop concepts related to integers, algebraic expressions, equations, and polynomials.***

2. Explain the approach you will use in the workshop.
   - ♦ Say: ***In this workshop, I, as the Facilitator, will act as the teacher. I'll use algebra tiles and Visual Aid transparencies to model the activities on an overhead projector. You, the participants, will act as students and experience the benefits of hands-on activities in math classrooms.***

3. Tell participants which activities you will use in the workshop.

## Workshop Opening: Examining the Algebra Tiles

Note:
*The Facilitator works with Visual Aid 1 and algebra tiles on the overhead projector while participants work in pairs, or in groups of 4, at their desks.*

Materials:

- *Visual Aid 1*
- *Algebra tiles for the overhead projector*
- *Algebra tiles for each group*
- *Newsprint or large paper sheets*

Note:
*When appropriate, pictures of models will appear in this column.*

Note:
*Choose **at least two activities** in addition to the workshop opening and closing.*

## Examine the Algebra Tiles (5 minutes)

4. Have participants spread the algebra tiles on their worktables and examine them.

5. Discuss the colors and shapes of the different algebra tiles.

   Ask questions such as:

   - Ask: ***What do you notice about all the negative tiles, –1, –x, and $–x^2$?***

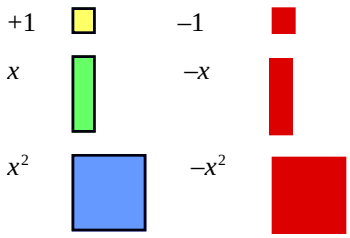
6. Have participants store the tiles in a corner of their worktables so they will have room to make models in the center of their tables.

7. Inform participants that they will use the newsprint to record their findings by drawing the models and writing the number sentences and equations they will create in the workshop activities.

Note

*Use a transparency of Visual Aid 1.*

*Place algebra tiles on the transparency.*




**Answer:**
**All the negative tiles are red.**

## Classroom Applications

Encourage participants to consult the Teacher's Editions for Prentice Hall *Middle Grades Math*, *Prentice Hall Algebra*, or *Prentice Hall Advanced Algebra* for classroom activities related to algebra tiles.

Algebra tiles help to develop concepts related to integers, algebraic expressions, equations, and polynomials. Point out that small yellow tiles (+1 tiles) are used for positive integers and small red tiles (–1 tiles) are used for negative integers.

# Activity 1: Adding Integers

## Adding Integers (10 minutes)

Note:
*The Facilitator works with algebra tiles on the overhead projector while participants work at their desks in pairs or in groups of 4.*

Materials:

- +1 tile 
- –1 tile 
- *Visual Aid 2*
- *Newsprint*

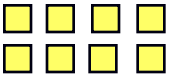
1. Use your algebra tiles on the overhead projector in the steps below to illustrate the addition of two positive numbers: 5 + 3

   ♦ Have participants show two groups of positive tiles. In one group, model +5. In the other group, model +3.

   ♦ Ask ***How can we model 5 + 3 with these tiles?***

   Elicit the fact that to add the two groups, they should be moved together.

   ♦ Ask: ***What number sentence describes the model?***

   **Answer: 5 + 3 = 8**

2. Since the sum of a number and its opposite is zero, together, a positive tile and a negative tile represent zero and are called a **zero pair**. Use your algebra tiles to model a zero pair on the overhead projector.

   ♦ Ask: ***What number sentence describes this zero pair?***

   **Answer: 1 + (–1) = 0**

3. Have participants model 3 + (–3) at their desks while you model the expression on the overhead projector. Model 3 with 3 yellow tiles and model –3 with 3 red tiles.

   ♦ Ask: ***What number sentence describes the model?***

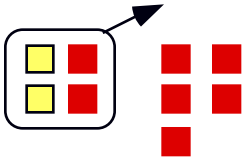
   

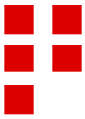
   **Answer: 3 + (–3) = 0**

4. Have participants model 2 + (– 7) at their desks while you model the expression on the overhead projector.
Model 2 with 2 positive tiles and –7 with 7 negative.

**Answer: 2 + (–7)**

- Ask: ***What expression does this model represent?***
- Have participants join the two groups of tiles. Match pairs of positive and negative tiles and remove them. Elicit the fact that you can remove zero pairs because their value is zero. Elicit the fact that you cannot form any more zero pairs because all yellow tiles have been used.





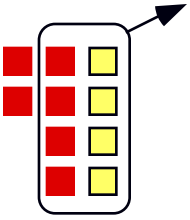
- Ask: ***After removing the zero pairs, what tiles are left?***

**Answer: 5 red tiles, representing –5**

- Ask: ***What number sentence describes the model?***

**Answer: 2 + –7 = –5**

5. Challenge participants to find another sum.
   - Ask: ***What is the sum of – 6 + 4?***
   - Have participants model the expression and find the sum. Their models should show 4 zero pairs and 2 red tiles, similar to what is shown at the right. On a piece of paper, have teams record their findings by drawing and coloring the algebra tiles and writing the number sentence.



**Answer: –6 + 4 = –2**

6. Place a transparency of Visual Aid 2 on the overhead projector. Cover the "Model/Answer" column. Have participants use algebra tiles to find the following sums. After each group has recorded its models and number sentences, uncover the "Model/Answer" column and have participants compare their models to those on the transparency.
   a. **4 + (– 9)**
   b. **– 3 + (– 8)**
   c. **9 + (– 3)**

Note:
*Use a transparency of Visual Aid 2*

*(5 minutes)*

## Classroom Applications

Encourage participants to consult the Teacher's Editions for Prentice Hall *Middle Grades Math*, *Prentice Hall Algebra*, or *Prentice Hall Advanced Algebra* for more activities related to integers.

Algebra tiles help to develop concepts related to integers, algebraic expressions, equations, and polynomials. Point out that small yellow tiles (+1 tiles) are used for positive integers and small red tiles (–1 tiles) are used for negative integers.

# Activity 2: Subtracting Integers

## Subtracting Integers (10 minutes)

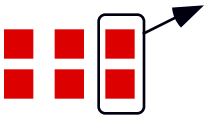
Note:
*The Facilitator uses algebra tiles on the overhead projector while participants work at their desks in pairs or in groups of 4.*

Materials:

- +1 tile
- –1 tile
- *Visual Aid 3*
- *Newsprint*

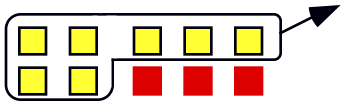
1. Use algebra tiles on the overhead projector in the steps below to illustrate the subtraction of two integers.
   - Say: ***We can model – 6 – (–2).***

     Start with 6 negative tiles. To subtract –2, remove 2 negative tiles.
   - Ask: ***What number sentence describes the model?***



**Answer: –6 – (–2) = –4**

2. Say: ***We can model 4 – 7.***
   - Have each group place four positive tiles in a group. Ask them to add zero pairs until they have seven positive tiles in the set.
   - Have the participants remove seven positive tiles.
   - Say*: **This is one form of subtraction.***
   - Ask: ***How many tiles are left?***
   - Ask: ***What number sentence describes the model***?



**Answer: Three red tiles, representing – 3.**

**Answer: 4 – 7 = – 3**

3. Place a transparency of Visual Aid 3 on the overhead projector. Cover the "Model/Answer" column. Have participants use algebra tiles for the following subtractions. After each group has recorded its models and number sentences, uncover the "Model/Answer" column.

   **a. 2 – 6 b. 2 + (–6) c. –3 – 8 d. –3 + (–8)**

Note:
*Use a transparency of Visual Aid 3*

*(5 minutes)*

4. Elicit discussion about the differences and sums in Visual Aid 3.
   - Say: ***Complete the following statement. To subtract an integer, you can add —.***

**Answer: its opposite**

One pre-service teacher commented about the class in which he was a substitute teacher:

> ***"They just don't get it unless they use the tiles. I've tried without them, and it just doesn't work."***

## Simplifying Algebraic Expressions (15 minutes)

1. Describe the algebra tiles the participants will use. Point out that negative tiles are red.

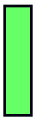
2. Write this expression on the chalkboard or on a transparency:
   - ♦ Write: ***5 + 4x***
   - ♦ Ask: ***How can we model this expression?***
   - ♦ Give participants an opportunity to respond. Then model the expression with the group.

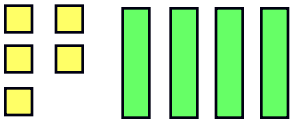
3. Write this expression on the chalkboard or on a transparency:
   - ♦ Write: ***3x + 2 – 4x – 5***
   - ♦ Say: ***Before we model this expression, remember that subtracting is the same as adding the opposite, so we can write the expression as 3x + 2 + (–4x) + (–5).***
   - ♦ Ask: ***How can we model this expression?***
   - ♦ Give participants an opportunity to respond. Then model the expression with the group.

## Activity 3: Simplifying Algebraic Expressions

Note:
*The Facilitator works with algebra tiles on the overhead projector while participants work at their desks in pairs or in groups of 4.*

Materials:

- +1 tile 
- –1 tile 
- *x* tile 
- –*x* tile 
- *Visual Aid 4*
- *Newsprint*



5 + 4*x*

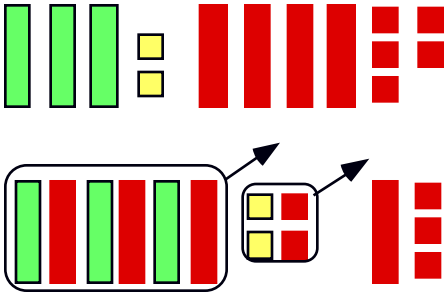
3*x* + 2 – 4*x* – 5

4. ♦ Ask: How can we simplify the expression?

   ♦ Elicit the fact that **simplifying means collecting like terms** (like tiles) by **using zero pairs.**

   ♦ Remove zero pairs of *x* tiles and zero pairs of integer tiles.

   ♦ Say: ***After we move aside the zero pairs, the simplified expression is left.***



**Answer:**
$3x + 2 - 4x - 5 = -x - 3$

5. Write this expression on the chalkboard or on a transparency:

   ♦ Write: **$-2x + 5 - 4x - 5$**

   ♦ Ask: ***How can we model this expression?***

   ♦ Give participants an opportunity to respond. Then model the expression with the group.

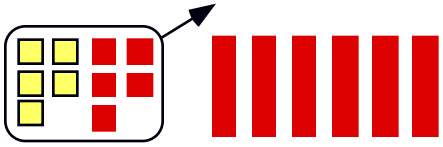
$-2x \quad + 5 \qquad - 4x \qquad - 5$

   ♦ Ask: ***How can we simplify the expression?***

   ♦ Repeat that **simplifying** means collecting like terms (like tiles) by **using zero pairs**.

   ♦ Remove the integer zero pairs.

   ♦ Ask: ***What is the simplified expression?***



**Answer:** **$-6x$**

6. Place a transparency of Visual Aid 4 on the overhead projector. Cover the "Model/Answer" column. Have participants use algebra tiles to model and simplify the following expressions. After each group has recorded its models and expressions, uncover the "Model/Answer" column.

   a. **$4x + 8 - 3x$**

   b. **$5x - 9 - 2 - 3x$**

   c. **$-3x + 7 + x - 6$**

Note:
*Use a transparency of Visual Aid 4*

*(5 minutes)*

## Classroom Applications

Encourage participants to consult the Teacher's Editions for Prentice Hall *Middle Grades Math*, *Prentice Hall Algebra*, or *Prentice Hall Advanced Algebra* for further classroom activities related to integers.

Algebra tiles can help students develop concepts related to integers, algebraic expressions, equations, and polynomials.

## Solving Linear Equations (15 minutes)

# Activity 4: Solving Linear Equations

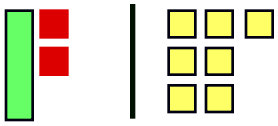
Note:
*The Facilitator works with algebra tiles on the overhead projector while participants work at their desks in pairs or in groups of 4.*

Materials:

- +1 tile 
- –1 tile 
- *x* tile 
- –*x* tile 
- *Visual Aid 5*
- *Newsprint*

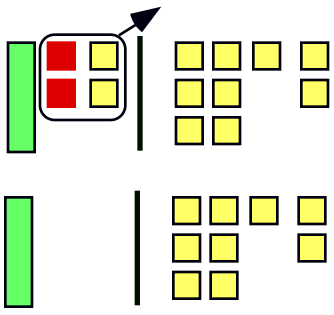
1. Describe the algebra tiles, shown at the right, that the participants will use. Point out that negative tiles are red.

2. Write this expression on the chalkboard or on a transparency:
   - Write: ***x* – 2 = 7**
   - Ask: ***How can we model this equation?***
   - Give participants an opportunity to respond. Then model the equation. Point out that just as the two sides of an equation are separated by an equals sign, the two parts of the equation are separated by the bar in the model.



*x* – 2 = 7

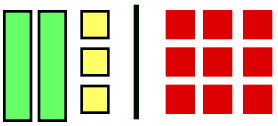
   - Ask: ***How can we use algebra tiles to solve this equation?***
   - Give participants an opportunity to respond.
   - Say***: To isolate the variable, that is, to get the x tile alone, add 2 positive tiles to each side of the equation.***
   - Remove the zero pairs to show the solution.
   - Ask: ***What is the solution?***



**Answer: *x* = 9**

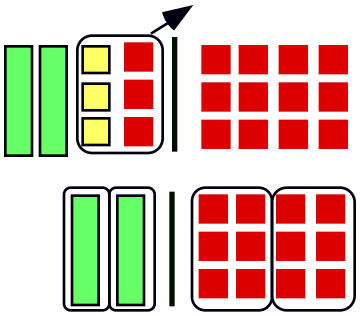
3. Write this expression on the chalkboard or on a transparency:

   - ♦ Write: **$2x + 3 = -9$**
   - ♦ Ask: ***How can we model and solve this equation?***
   - ♦ Give participants an opportunity to respond. Then manipulate the tiles while you explain each step.



$2x + 3 = -9$

   - ♦ Add three negative tiles to each side to create zero pairs on the side with the *x*-tiles.
   - ♦ Remove zero pairs to show $2x = -12$.

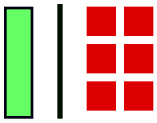


$2x = -12$

   - ♦ Say: ***We want to get x alone for a solution. First, we can make two groups of equal numbers of tiles on each side of the bar. Then we can remove one set of the tiles from each side of the bar. Remember that whatever we do to one side of an equation, we must do to the other side.***

   **$2x \div 2 = -12 \div 2$**

   - ♦ Ask: ***What is the solution?***



**Answer: $x = -6$**

4. Place a transparency of Visual Aid 5A on the overhead projector. Cover the "Model/Answer" column. Have participants use algebra tiles to solve the following equation. After each group has recorded its model and solution, uncover the "Model/Answer" column.

   - ♦ Say: ***Solve this equation:*** **$3x - 2 = 4$**

Note:
*Use a transparency of Visual Aid 5A.*

**Answer: $x = 2$**

5. Place a transparency of Visual Aid 5B on the overhead projector. Cover the "Model/Answer" column. Have participants use algebra tiles to solve the following equations. After each group has recorded its models and solutions, uncover the "Model/Answer" column.

   - ♦ Say: ***Solve this equation:*** **$5x + 6 = -4$**

Note:
*Use a transparency of Visual Aid 5B.*

**Answer: $x = -2$**

In this activity you will challenge participants to work in cooperative groups to analyze a word problem, determine how they can use algebra tiles to model the problem, and how they can use algebra tiles to solve the problem.

## Using Algebra Tiles to Solve a Word Problem (10 minutes)

1. Read this problem to participants or display a transparency of Visual Aid 6A.

   - **A total of 38 students from two schools attended a mathematics tournament. Six students from Jefferson High School rode bicycles. The rest were transported in three cars. Seven students from Lincoln Middle School rode bicycles. The rest were transported in two cars. If all the cars transported the same number of students, how many students rode in each car?**

2. Tell participants they will work together in their groups for 5 minutes to:
   - Write an equation for the problem.
   - Model the problem with algebra tiles.
   - Solve the problem.

3. After 5 minutes display a transparency of Visual Aid 6B on the overhead projector. Discuss the equation, the models, and the solution with participants.

## Classroom Applications

Encourage participants to consult the Teacher's Editions for Prentice Hall *Middle Grades Math*, *Prentice Hall Algebra*, or *Prentice Hall Advanced Algebra* for further classroom activities related to problem solving.

## Activity 5: Solving Word Problems with Algebra Tiles

Materials:

- *Each group of 4 participants needs enough algebra tiles to represent 51 unit tiles and 5 x-tiles.*
- *Visual Aids 6A, 6B*

Note:
*Use a transparency of Visual Aid 6A to present the problem.*

Note:
*Allow 5 minutes.*
*Remind students to record their models and equations.*

Equation:
$3x + 6 + 2x + 7 = 38$

Note:
*Uncover the transparency for Visual Aid 6B row by row.*

There are many more activities that can be done with algebra tiles beyond representing algebraic expressions and linear equations. Algebra tiles can be used to represent polynomials and operations with polynomials. The tiles can also be used to factor trinomials and to solve quadratic equations.

## Modeling Polynomials (5 minutes)

1. Review the meaning of each algebra tile with participants.

2. Place the tiles to the right on the overhead projector.
   - ♦ Ask: ***What expression does this model represent?***

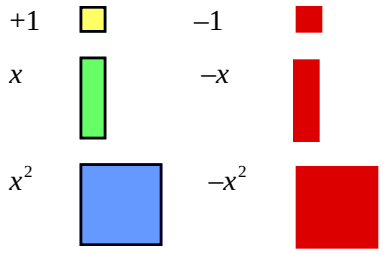
3. Challenge participants to build a model of the expression:
   - ♦ Say: ***Build a model for this expression:***
     **$2x^2 - 2x - 3$.**

## Adding Polynomials (5 minutes)

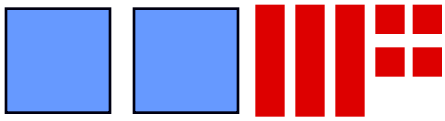
1. Remind participants that they can model **addition of polynomials** by modeling the two polynomials, joining them, and removing zero pairs.
   - ♦ Ask: ***How can we model this addition?***
     **$2x^2 + 3x + 5$ and $x^2 - 2x - 3$**
   - ♦ Accept all reasonable answers. Then model the addition with tiles on the overhead projector. Call attention to the zero pairs.
     - ♦ ***Ask:*** What is the sum?

## Activity 6: Adding and Subtracting Polynomials

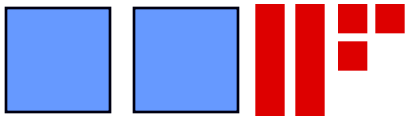
Materials:



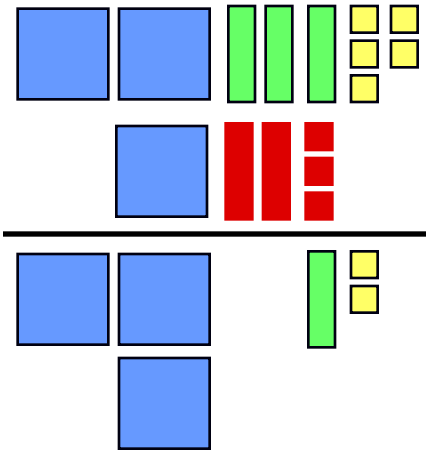

- *Visual Aids 1, 7A, 7B*
- *Newsprint*



Answer: $2x^2 - 3x - 4$



$2x^2 - 2x - 3$



**Answer: $3x^2 + x + 2$**

2. Place a transparency of Visual Aid 7A on the overhead projector. Cover the "Model/Answer" section. Have participants use algebra tiles to add the following polynomials. After each group has recorded its models and solutions, uncover the "Model/Answer" section.

   - *Say:* ***Find the sum:***
     **$(3x^2 + 2x - 4) + (-2x^2 + x - 3)$**

Note:
*Use a transparency of Visual Aid 7A.*

**Answer:**
**$x^2 + 3x - 7$**

## Subtracting Polynomials (5 minutes)

1. Remind participants that they can model **subtraction of a polynomial** by adding the model of the inverse of the polynomial to be subtracted to the model of the first polynomial.

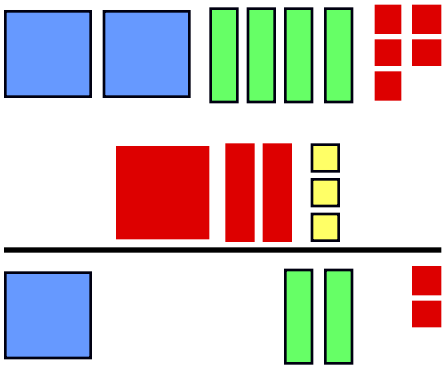
   - Ask: ***How can we model this subtraction?***
     **$2x^2 + 4x - 5 - (x^2 + 2x - 3)$**
   - Accept all reasonable answers. Then model the addition with tiles on the overhead projector.

Note:
*By the time students get to polynomials, they should know that subtraction is adding the inverse, or opposite, number.*

2. Build a model for $2x^2 + 4x - 5$.

3. Subtract $x^2 + 2x - 3$ by adding its opposite, that is $-x^2 - 2x + 3$. Model this expression.

   - Combine models and remove zero pairs to model the result.
   - Ask: ***What expression does the resulting model represent?***



**Answer: $x^2 + 2x - 2$**

4. Place a transparency of Visual Aid 7B on the overhead projector. Cover the "Model/Answer" section. Have participants use algebra tiles to subtract the following polynomials. After each group has recorded its models and solutions, uncover the "Model/Answer" section.

   - Say: ***Find the difference:***
     **$(2x^2 + 2x - 1) - (x^2 - x + 3)$**

Note:
*Use a transparency of Visual Aid 7B.*

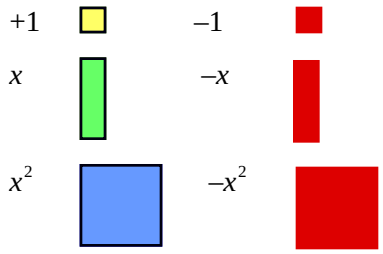
**Answer: $x^2 + 3x - 4$**

In this activity participants will use rectangular arrays to model the product of two polynomials. You can relate the multiplication of polynomials to a basic whole-number multiplication table. Place the first factor vertically under the multiplication dot. Place the second factor horizontally to the right of the multiplication dot.

# Activity 7: Multiplying Polynomials

## Multiplying Polynomials (15 minutes)

Materials:




- *Visual Aids 1, 8A, 8B*
- *Newsprint*

1. Review the meaning of each algebra tile with participants.

2. Ask participants how the table on Visual Aid 8A shows basic multiplication facts.

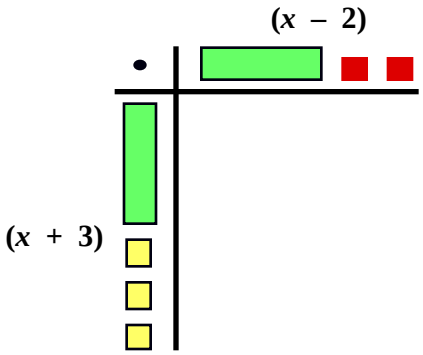
   - Ask: ***How can we find the product 3 • 4 in this table?***
   - Ask: ***What is the product of 3 • 4?***

Note:
*Use Visual Aid 8A.*

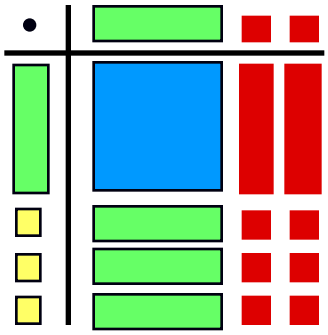
**Answer:**
**Find the entry to the right of 3 and under 4.**
**3 • 4 = 12**

3. Help participants relate a basic multiplication table to a rectangular array to model the multiplication of polynomials.

   - Ask: ***How can we model the multiplication of*** **$(x + 3)(x - 2)$?**
   - Elicit the fact that algebra tiles that represent the first factor $(x + 3)$ are placed on the vertical axis and algebra tiles that represent the second factor $(x - 2)$ are placed on the horizontal axis.



- Ask: ***How can we model*** **$(x + 3)(x - 2)$?**
- Give participants a chance to respond and model the multiplication with algebra tiles on the overhead projector.



**$(x + 3)(x - 2)$**

- Ask: ***What is the product of*** **$(x + 3)(x - 2)$?**

**Answer: $x^2 + 3x - 2x - 6 = x^2 + x - 6$**

4. Place a transparency of Visual Aid 8A on the overhead projector. Cover the "Model/Answer" column. Have participants use algebra tiles to model the following multiplications of polynomials. After each group has recorded its models and solutions, uncover the "Model/Answer" column and discuss the groups' findings.

   **$(x - 1)(x - 4)$**

Note:
*Use a transparency of Visual Aid 8A.*

**Answer: $x^2 - 5x + 4$**

5. Place a transparency of Visual Aid 8B on the overhead projector. Cover the "Model/Answer" column. Have participants use algebra tiles to model the following multiplications of polynomials. After each group has recorded its models and solutions, uncover the "Model/Answer" column and discuss the groups' findings.

   **$(-2x + 2)(x - 3)$**

Note:
*Use a transparency of Visual Aid 8B.*

**Answer: $-2x^2 + 8x - 6$**

## Classroom Applications

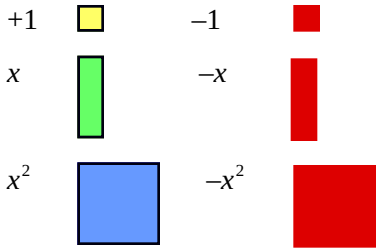
Encourage participants to consult the Teacher's Editions for Prentice Hall *Middle Grades Math*, *Prentice Hall Algebra*, or *Prentice Hall Advanced Algebra* for further classroom activities related to polynomials.

In this activity participants will use rectangular arrays to model factoring polynomials. Relate factoring polynomials to Activity 7, Multiplying Polynomials. The first factor is placed vertically under the multiplication dot. The second factor is placed horizontally to the right of the multiplication dot.

# Activity 8: Factoring Polynomials

## Factoring Polynomials (15 minutes)

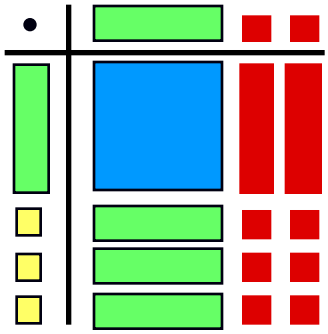
Materials:




- *Visual Aids 1, 9A, 9B*
- *Newsprint*

1. Review the meaning of each algebra tile with participants.

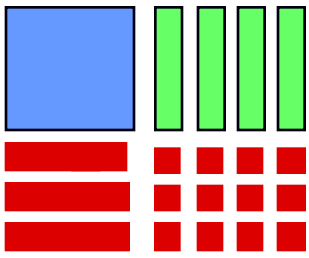
2. Briefly review Multiplying Polynomials.
   - Ask: ***How can we model the multiplication of $(x + 3)(x – 2)$?***
   - Elicit the fact that algebra tiles that represent the first factor ($x + 3$) are placed on the vertical axis and algebra tiles that represent the second factor ($x – 2$) are placed on the horizontal axis.
   - Give participants a chance to respond. Then model the multiplication with algebra tiles on the overhead projector.



**$(x + 3)(x – 2) = x^2 + x – 6$**

3. Show the model at the right on the overhead projector.

Note*: Use a transparency of Visual Aid 8A or model the expression with algebra tiles on the overhead projector.*

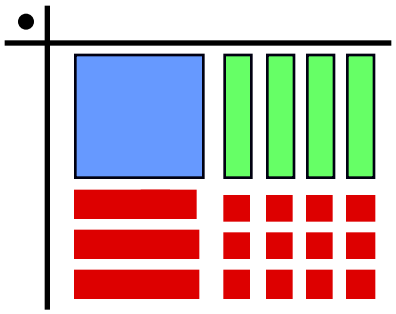


   - Call attention to the fact that the model shows a rectangular array with the tiles arranged in descending order.
   - Ask: ***What expression does this model represent?***

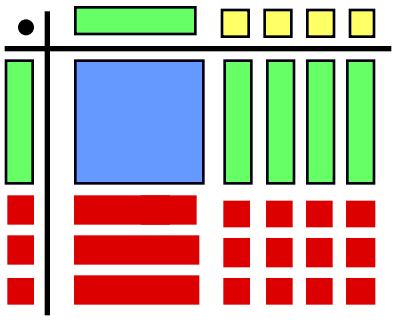
**Answer: $x^2 + 4x – 3x – 12$**

4. Challenge participants to suggest how the expression can be factored by using algebra tiles. Give participants a chance to respond and then demonstrate on the overhead projector how to factor a polynomial.

   ♦ Build an axis around the rectangle.



   ♦ To factor the polynomial, find the dimensions of the rectangle. Determine which tiles should be placed on the horizontal axis and which tiles should be placed on the vertical axis. Call attention to the fact that all negative tiles should be placed on the same axis.



   ♦ Ask: ***What expression does the resulting model represent?***

**Answer: $(x - 3)(x + 4)$**

5. Place a transparency of Visual Aid 9A on the overhead projector. Cover the "Model/Answer" column. Have participants use algebra tiles to model the factoring of the following trinomial. After each group has recorded its model and solution, uncover the "Model/Answer" column and discuss the groups' findings.

   ♦ Factor $x^2 + 5x + 6$.

Note:
*Use a transparency of Visual Aid 9A.*

**Answer:**
**$x^2 + 5x + 6 =$**
**$(x + 2)(x + 3)$**

6. Place a transparency of Visual Aid 9B on the overhead projector. Cover the "Model/Answer" column. Have participants use algebra tiles to model the factoring of the following trinomial. After each group has recorded its model and solution, uncover the "Model/Answer" column and discuss the groups' findings.

   ♦ Factor $x^2 - 7x + 12$.

Note:
*Use a transparency of Visual Aid 9B.*

**Answer:**
**$x^2 - 7x + 12 =$**
**$(x - 4)(x + 3)$**

## Classroom Applications

Encourage participants to consult the Teacher's Editions for Prentice Hall *Middle Grades Math*, *Prentice Hall Algebra*, or *Prentice Hall Advanced Algebra* for further classroom activities related to factoring polynomials.

While a representative from each group displays the pages with the group's recordings, you may wish to collect the algebra tiles and other materials that must be returned.

## Closing Activities (10 minutes)

1. Have a representative from each group display around the room the recordings (drawings, number sentences, equations) from that group.

2. Recap the activities that have been presented and end by having participants discuss what they have learned and how they can use it.

   - Ask: ***How can the activities that we worked on today have an impact on students in your classroom from now on?***

3. Stress with participants the importance of continuing to experiment with algebra tiles in their classrooms.

   - Say: ***Remember, algebra tiles can help students develop concepts related to integers, algebraic expressions, equations, and polynomials.***

4. You may wish to refer your group to any worthwhile articles, books, or Web sites with which you are familiar.

5. Take any questions that remain.

6. Collect all algebra tiles and other materials that have been distributed.

7. You may wish to close with the quotation that introduced this professional development workshop:

   - Say: ***Remember:***

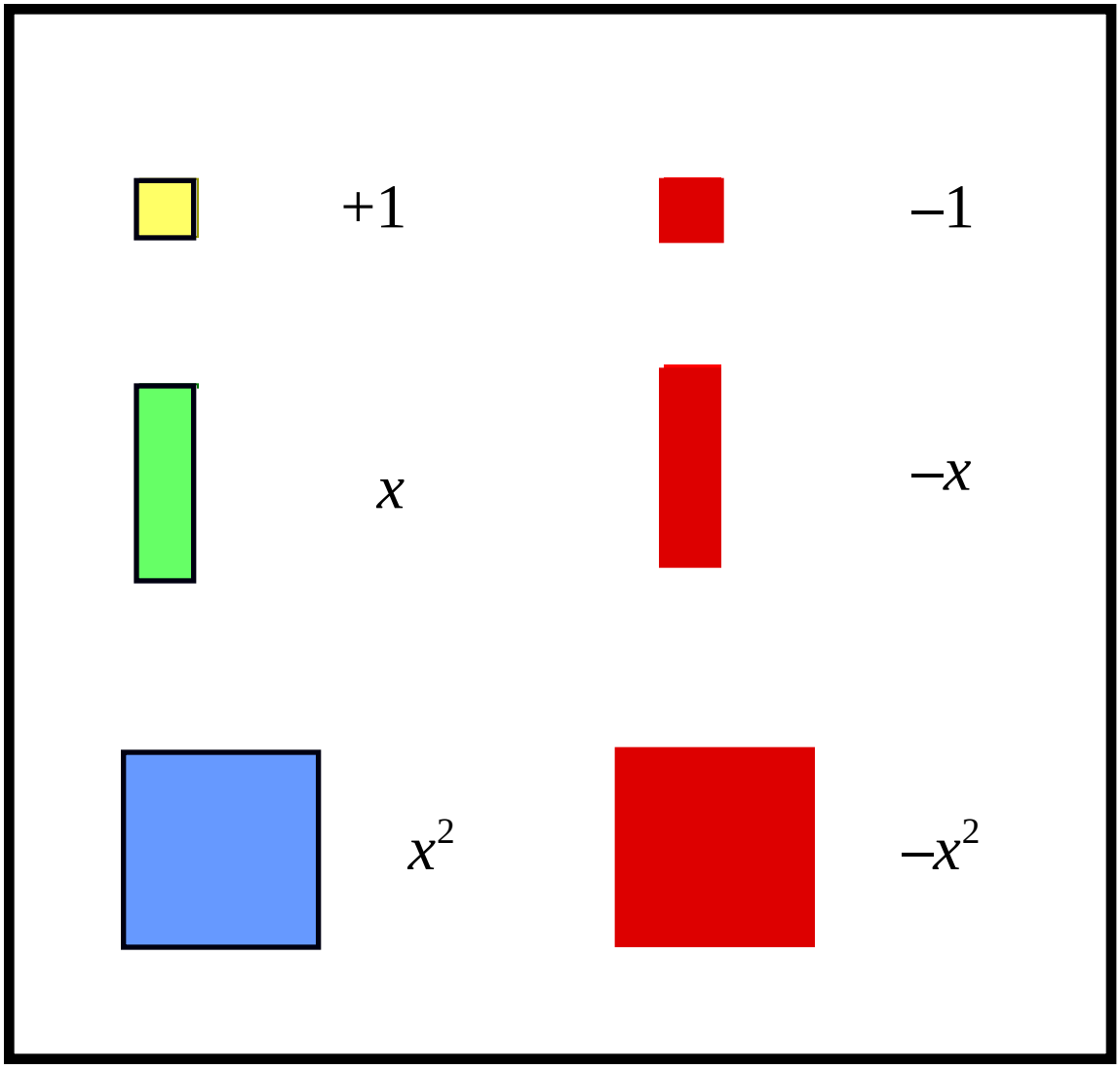
     ***When I listen, I hear.***
     ***When I see, I remember.***
     ***But when I do, then I understand.***

## Workshop Closing: Recapping the Activities

Note:
*For more professional development information and training materials, visit the Prentice Hall site:*

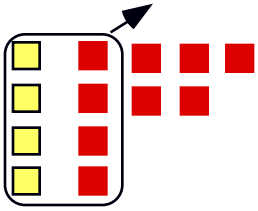
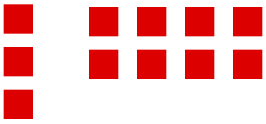
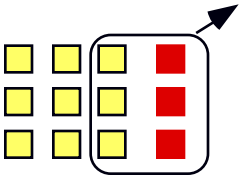
**www.phschool.com**

# VISUAL AID 1
# WORKING WITH ALGEBRA TILES



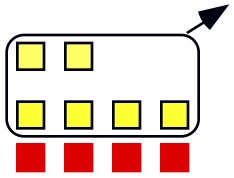
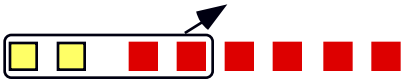
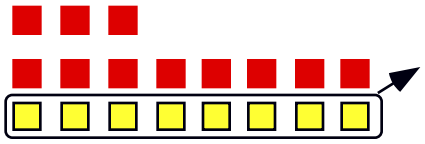
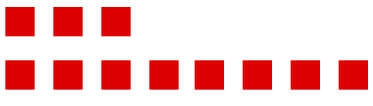
# VISUAL AID 2
# ADDING INTEGERS

Use algebra tiles to find the following sums.
Write a number sentence for each.
Record your models and number sentences.

| Expression | Model/Answer |
|---|---|
| $4 + (-9)$ |  $4 + (-9) = -5$ |
| $-3 + (-8)$ |  $-3 + (-8) = -11$ |
| $9 + (-3)$ |  $9 + (-3) = 6$ |

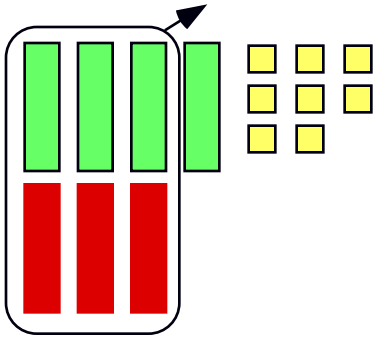
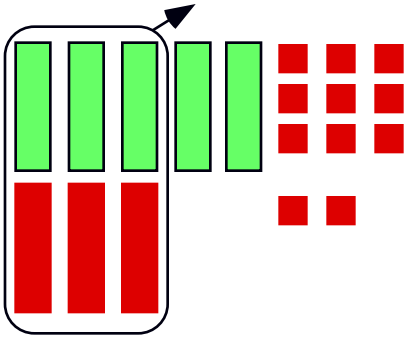
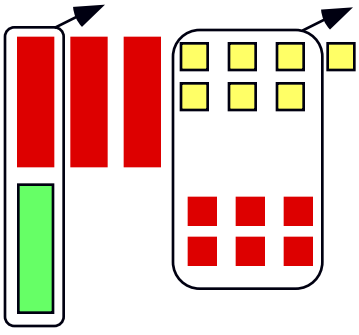
# VISUAL AID 3
## SUBTRACTING INTEGERS

Model these differences and sums with algebra tiles. Write a number sentence for each. Record your models and number sentences.

| Expression | Model/Answer |
|---|---|
| 2 – 6 |  **2 – 6 = –4** |
| 2 + (–6) |  **2 + (– 6) = –4** |
| –3 – 8 |  **–3 – 8 = –11** |
| –3 + (–8) |  **–3 + (–8) = –11** |

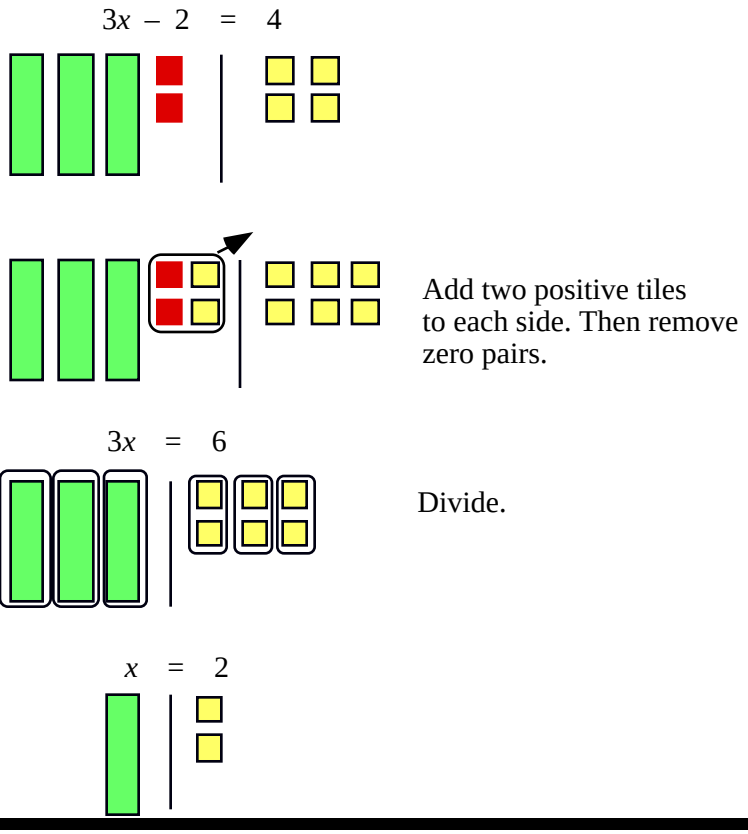
# VISUAL AID 4
# SIMPLIFYING ALGEBRAIC EXPRESSIONS

Simplify these expressions.
Record your findings.

| Expression | Model/Answer |
|---|---|
| $4x + 8 - 3x$ |  $4x + 8 - 3x = x + 8$ |
| $5x - 9 - 2 - 3x$ |  $5x - 9 - 2 - 3x = 2x - 11$ |
| $-3x + 7 + x - 6$ |  $-3x + 7 + x - 6 = -2x + 1$ |

# VISUAL AID 5A
# SOLVING LINEAR EQUATIONS

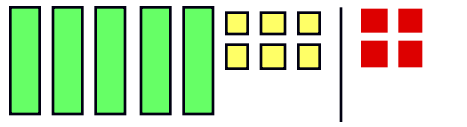
Model this equation with algebra tiles.
Use the model to solve the equation.
Record your findings.

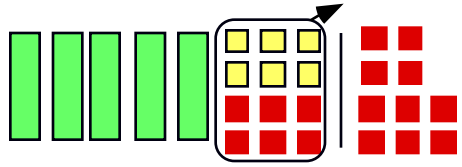
| Equation | Model/Answer |
|---|---|
| $3x - 2 = 4$ |  |

# VISUAL AID 5B
# SOLVING LINEAR EQUATIONS

Model this equation with algebra tiles.
Use the model to solve the equation.
Record your findings.

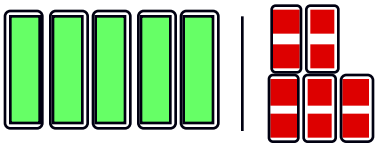
$5x + 6 = -4$





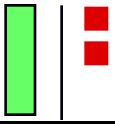
Add six negative tiles to each side. Then remove zero pairs.

$5x = -10$



Divide.

$x = -2$



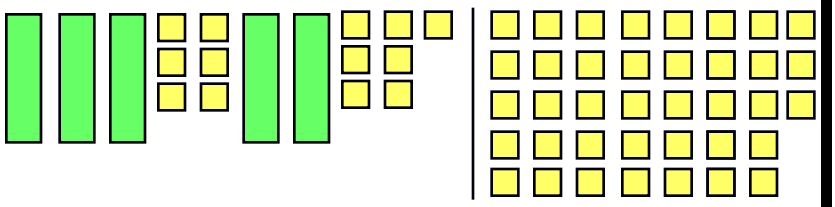
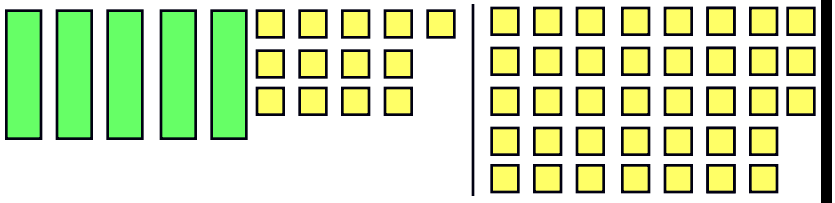
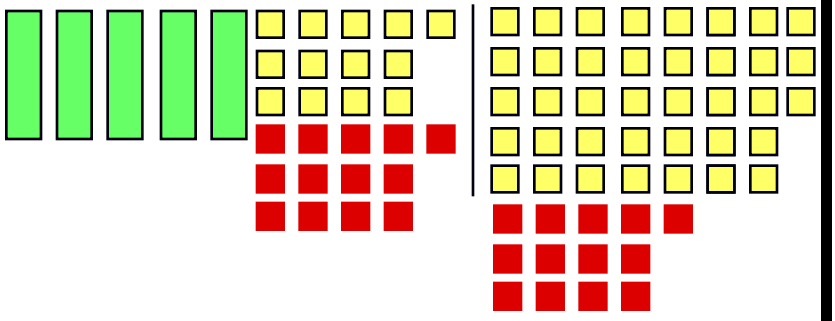
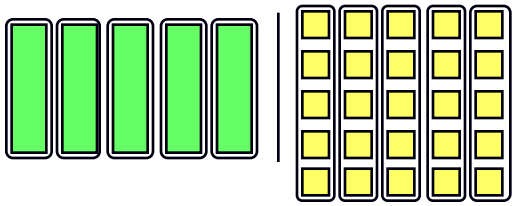
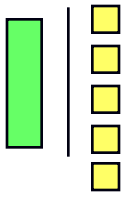
# VISUAL AID 6A
# SOLVING WORD PROBLEMS

Model the word problem with algebra tiles.
Use the model to solve the problem.
Record your findings.

**A total of 38 students from two schools attended a mathematics tournament. Six students from Jefferson High School rode bicycles. The rest were transported in three cars. Seven students from Lincoln Middle School rode bicycles. The rest were transported in two cars. If all the cars transported the same number of students, how many students rode in each car?**

# VISUAL AID 6B
# SOLVING WORD PROBLEMS

Use the model for Visual Aid 6A.
Solve the word problem. Record your findings.

| | |
|---|---|
| **$3x + 6 + 2x + 7 = 38$** |  |
| **Simplify the left side of the equation.** |  |
| **Add –13 to each side of the equation to form zero pairs.**<br><br>**Remove zero pairs.** |  |
| **$5x = 25$** |  |
| ***x* = 5**<br><br>**Five students rode in each car.** |  |

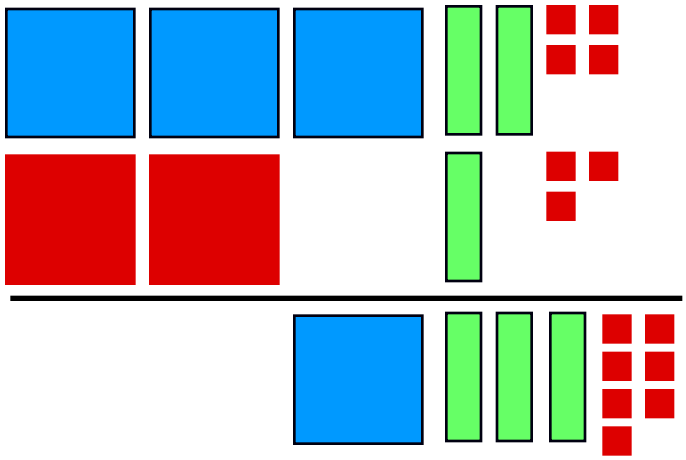
# VISUAL AID 7A
# ADDING POLYNOMIALS

Use algebra tiles to find the following sum.
Write an equation for the addition.
Record your model and equation.

## Expression

$$(3x^2 + 2x - 4) + (-2x^2 + x - 3)$$

## Model/Answer



$$(3x^2 + 2x - 4) + (-2x^2 + x - 3) = x^2 + 3x - 7$$

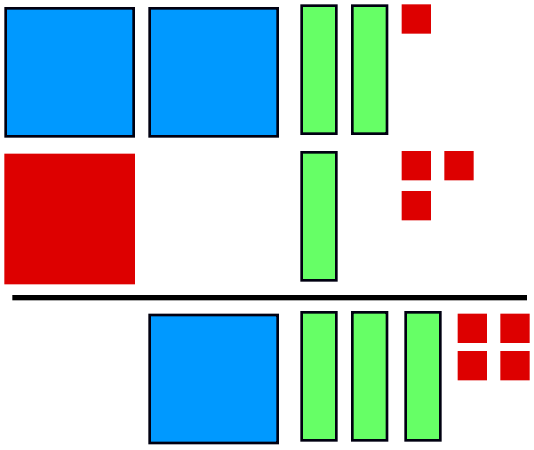
# VISUAL AID 7B
# SUBTRACTING POLYNOMIALS

Use algebra tiles to subtract.
Write an equation for the subtraction.
Record your model and equation.
*Remember: Subtraction is adding the inverse number.*

**Expression**

$$(2x^2 + 2x - 1) - (x^2 - x + 3)$$

**Model/Answer**



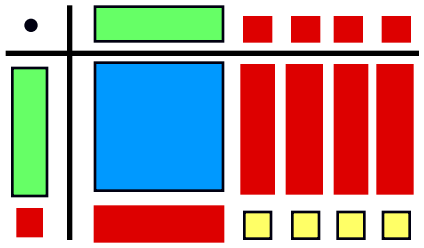
$$(2x^2 + 2x - 1) - (x^2 - x + 3) = x^2 + 3x - 4$$

# VISUAL AID 8A
# MULTIPLYING POLYNOMIALS

| • | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 10 |
| 3 | 5 | 9 | 12 | 15 |
| 4 | 8 | 12 | 16 | 20 |
| 5 | 10 | 15 | 20 | 25 |

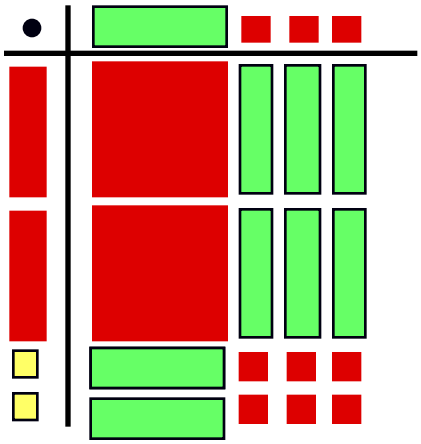
$3 \cdot 4 = 12$

Model this multiplication with algebra tiles. Record your findings.

| Multiply | Model/Answer |
|---|---|
| $(x - 1)(x - 4)$ |  $x^2 - 5x + 4$ |

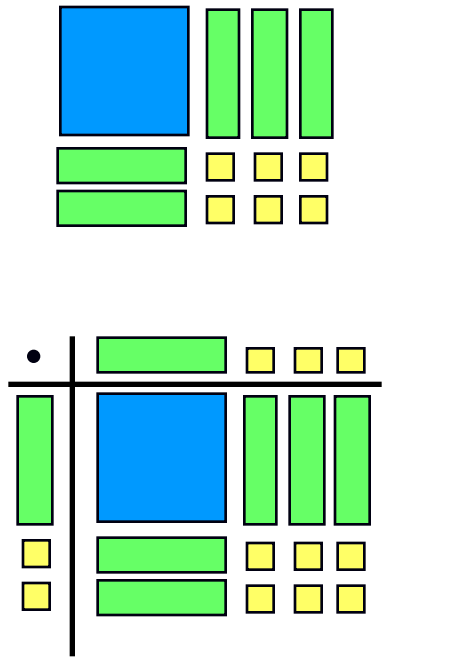
# VISUAL AID 8B
# MULTIPLYING POLYNOMIALS

Model this multiplication with algebra tiles. Record your findings.

| | |
|---|---|
| $(-2x + 2)(x - 3)$ |  Answer: $-2x^2 + 8x - 6$ |

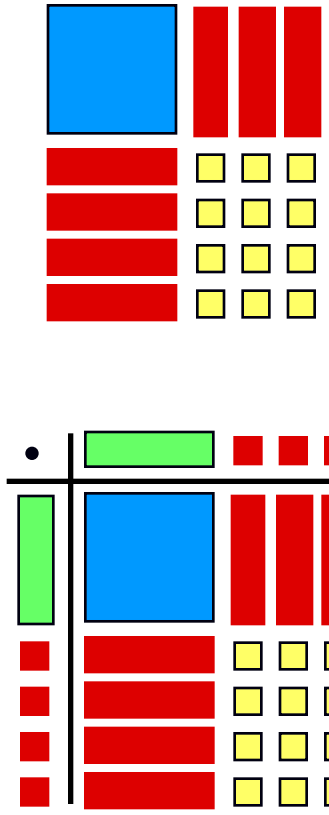
# VISUAL AID 9A
# FACTORING POLYNOMIALS

Use algebra tiles to factor this trinomial. Record your findings.

| Factor | Model/Answer |
|---|---|
| $x^2 + 5x + 6$ |  Answer: $x^2 + 5x + 6 = (x + 2)(x + 3)$ |

# VISUAL AID 9B

Use algebra tiles to factor this trinomial. Record your findings.

| Factor | Model/Answer |
|---|---|
| $x^2 - 7x + 12$ |  Answer: $x^2 - 7x + 12 = (x - 4)(x - 3)$ |